Flying Qualities: an Integral Part of a Stability Augmentation System

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During the conception, development, and flight testing of a stability augmentation system designed to improve the structural response characteristics of the B-52, it was necessary to provide flying qualities criteria and to conduct piloted simulator studies to assure that the B-52 flying qualities were improved or maintained. The fixed-base simulator studies utilized a point light projection display system and a B-52 cockpit display and included the effect of random turbulence. The results of the studies conducted for the lateral-directional axis are presented. In the piloted simulation study the Dutch roll frequency and damping were varied while other pertinent lateral-directional parameters were maintained constant. It was found that the Dutch roll damping required to provide satisfactory flying qualities in turbulence was significantly greater than the damping required in essentially calm air. Improvements in the handling qualities as verified by flight test are also presented.

Nomenclature

C_L = airplane lift coefficient
$C_{l\beta}$ = coefficient of roll moment change due to variation of sideslip angle, per rad
$C_{l\delta_a}$ = coefficient of roll moment change due to lateral control deflection, per rad
C_{l_p} = coefficient of roll moment change due to variation in roll rate, per rad
C_{l_r} = coefficient of roll moment change due to variation in yaw rate, per rad
$C_{n\beta}$ = coefficient of yawing moment change due to variation in sideslip angle, per rad
$C_{n\delta_a}$ = coefficient of yawing moment change due to lateral control deflection, per rad
C_{n_p} = $\partial C_n/\partial(pb/2V)$ = coefficient of yawing moment change due to variation in roll rate, per rad
C_{n_r} = coefficient of yawing moment change due to variation in yaw rate, per rad
$C_{y\beta}$ = coefficient of side force change due to variation of sideslip angle, per rad
I_{xx} = moment of inertia about stability roll axis, slug-ft^2
I_{zz} = moment of inertia about stability yaw axis, slug-ft^2
P_{ss} = steady-state roll rate, rad/sec
T_R = roll time constant, sec
T_S = spiral time constant, sec
β = angle of sideslip, positive nose left, rad
δ_a = aileron deflection, rad
δ_d = Dutch roll mode damping ratio
δ_ϕ = damping ratio of numerator oscillatory mode in ϕ/δ_a transfer function
ϕ = roll angle, positive for right wing down, rad
ω_d = Dutch roll undamped frequency, rad/sec
ω_ϕ = undamped natural frequency of numerator oscillatory mode in ϕ/δ_a transfer function, rad/sec

Introduction

An Air Force sponsored study was conducted by The Boeing Company during 1964 and 1965 to determine the changes to the B-52 flight control and stability augmentation systems (SAS) that would provide meaningful improvements in the airplane structural life and the aerodynamic and structural stability in severe turbulence. This study was conducted as a part of a continuing program to provide B-52 fleet longevity and effectiveness to meet Air Force requirements during the next decade. The results of the study program, available in August 1965, indicated that significant reductions in structural fatigue and peak loads could be expected if an advanced stability augmentation system were installed on the B-52.

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Development of the prototype stability augmentation system was accomplished during 1966 and 1967. Reference 3 summarizes SAS analyses and synthesis. Structural analyses conducted and a summary of the analytical results obtained are presented in Ref. 4. The system selected for development included both pitch and yaw stability augmentation.

A prototype model of the advanced SAS was designed, fabricated, and installed on a B-52H flight test airplane. Flight testing of the prototype SAS was completed in 1967 to optimize and demonstrate the SAS performance in terms of reducing peak structural loads and fatigue damage rates. The optimization was accomplished within the boundaries of adequate handling qualities and dynamic stability of the airplane.

Comparisons of the analytic and flight test load reduction are given in Ref. 1. The handling qualities aspects of the program are discussed in the following paragraphs.

Handling Qualities Considerations

During the initial studies, piloted simulation studies were conducted to evaluate the handling qualities and to determine the effect of pilot inputs on airframe loads. Handling qualities were evaluated at high-altitude cruise, low-altitude high-speed, landing approach, and aerial refueling conditions.

The essential feature of the piloted simulator studies was the assessment of effects of the various stability augmentation system concepts on handling qualities. Effects on over-all system performance were determined by means of pilot opinion ratings, and root mean square sideslip, roll angle, and angle-of-attack measurements. These parameters are indicative, in a qualitative sense, of airframe loading that is experienced during flight.

The configurations tested included: 1) magnetic yaw damper (MYD), 2) electronic yaw damper (EYD), 3) improved yaw SAS, and 4) improved yaw SAS plus roll SAS. Effects of turbulence were investigated by varying the turbulence input intensity.

The results of the initial studies indicated that the current flying qualities criteria were not adequate and that additional criteria would need to be developed. The principal element of the criteria development studies was concerned with the Dutch roll mode, with emphasis placed on the low-frequency spectrum. Also, during this development phase the proposed stability augmentation systems were evaluated with regard to the proposed criteria, and with past knowledge, to assure that the SAS's would provide good flying qualities. The tasks utilized for the simulation and performance measures were similar to those employed during the initial studies.

The flying qualities of the airplane with the SAS incorporated were continuously evaluated during the flight test program. Quantitative data pertaining to the airplane static and dynamic stability and control characteristics were also obtained and compared to the criteria. The data were also compared to the present fleet configuration in order to ascertain more directly the benefits of the SAS with respect to the current fleet airplanes.

Piloted Simulation Test Procedure

Basic ground rules were observed in the conduct of the tests, as follows:

1) Test conditions were run in a random fashion with respect to SAS configuration, turbulence intensity, and parameter variation. This was done to minimize learning-curve effects and to insure a different test sequence for each pilot.

2) A pilot was never informed as to what configuration he was testing at any time.

3) Results were not discussed with the pilots during the entire program.

4) Pilots were informed of the purpose of the test and details of the task to assure understanding of test requirements.

5) The pilots were given a warmup period in the simulator to minimize learning effects.

6) Session times were limited to avoid pilot fatigue.

7) Pilots comments, solicited and unsolicited, were recorded by test engineers and by voice recordings.

Analog Computer Program

The analog computer program utilized for the simulations included the cross-coupling effects, inertial and aerodynamic, between the longitudinal and lateral-directional axes, as well as the primary and secondary control system characteristics. The dynamics of the simulated unaugmented airplane were compared to flight test results of the unaugmented airplane to assure that the airplane was accurately simulated. Important nonlinearities, such as the spoiler characteristics, and authority limits, such as maximum rudder due to SAS and rudder blowdown, were also included in the program to improve the simulation. The program provided for large bank angles and large heading changes and small changes in pitch attitude. The random turbulence inputs in the vertical (angle-of-attack) directional (angle-of-sideslip) and the asymmetrical vertical (roll rate) were obtained using three white noise generators and appropriate filters. A switching network was used to facilitate changes between runs and to avoid mechanical errors by the computer operators.

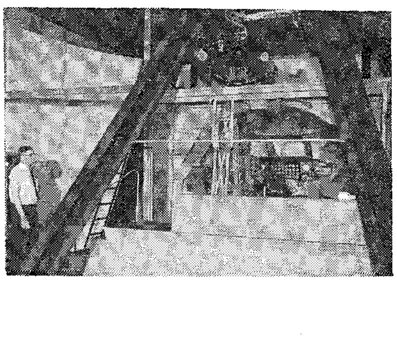

Fig. 1 Point light source simulator.

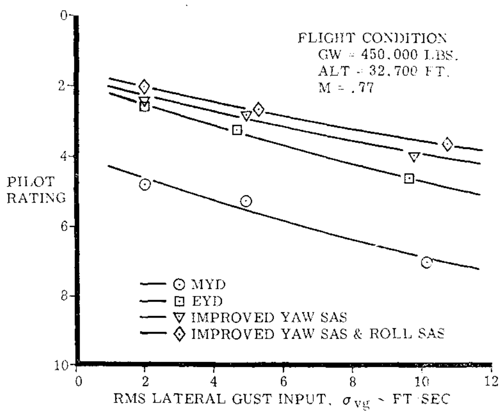

Fig. 2 Pilot evaluation of stability augmentation systems.

Point Light Source Display

The point light source display used for the flying qualities studies utilized a small, brilliant source of light reflecting from a painted mirror surface and shining through a color transparency to produce a wide-angle display simulating the world outside the pilot's windscreen. The screen was a 30-ft-diam fiberglass bowl covering 180° in azimuth and 120° in elevation.

The projection mechanism provides for all three degrees of angular freedom with angular response rates exceeding those of the simulated aircraft. The presentation represented a brilliant clear blue sky over a desolate desert with a well-defined horizon.

The peripheral vision characteristic of this simulator was particularly beneficial for the Dutch roll criteria development studies. Augmented by primary flight instruments, the simulator provided a highly effective set of visual cues for high-altitude flight. A photograph of the simulator installation is given in Fig. 1.

Piloting Tasks and Flight Condition Description

The program employed a comparatively small number of skilled flight test pilots and engineering pilots, all of whom were thoroughly familiar with the basic B-52 characteristics and mission requirements. The piloted simulation studies utilized various fixed-base simulation facilities to simulate four separate phases of the B-52 operation: 1) high-altitude cruise speed, 2) low-altitude high speed, 3) low-altitude power approach, and 4) high-altitude aerial refueling. During this program effects of yaw, roll, and pitch stability augmentation systems were investigated. Results of the various flight conditions were similar; therefore, only the results for the high-altitude condition are included in this paper.

The instructions to the pilots were as follows: "Fly specified heading, airspeed, and altitude, maintaining wings level for approximately two minutes. Execute a heading change of 45 degrees to the left, rolling onto the proper heading. Return to original heading. Fly original heading, airspeed, and altitude, maintaining wings level for approximately two minutes." The flight condition utilized for the simulation data described herein was as follows: high-altitude cruise; $GW = 450{,}000$ lb, $h = 32{,}700$ ft, $M = 0.77$, $V_e = 275$ knots.

Initial Comparison Study Results

Pilot ratings (Cooper rating scale, Table 1) obtained during this testing showed definite improvement due to the EYD compared to the MYD, due to improved yaw SAS compared to EYD, and due to the roll SAS. These results are shown in Fig. 2 in the form of pilot ratings vs rms lateral gust input intensity. The data shown are averages for the seven pilots who participated in the program. A general trend toward poorer rat-

Table 1 Pilot opinion rating system

Adjective rating	Numerical rating	Description	Primary mission accomplished	Can be landed
	1	Excellent, includes optimum	Yes	Yes
Satisfactory	2	Good, pleasant to fly	Yes	Yes
	3	Satisfactory, but with some mildly unpleasant characteristics	Yes	Yes
	4	Acceptable, but with unpleasant characteristics	Yes	Yes
Unsatisfactory	5	Unacceptable for normal operation	Doubtful	Yes
	6	Acceptable for emergency condition only[a]	Doubtful	Yes
	7	Unacceptable even for emergency conditions[a]	No	Doubtful
Unacceptable	8	Unacceptable—dangerous	No	No
	9	Unacceptable—uncontrollable	No	No
Catastrophic	10	Motions possibly violent enough to prevent pilot escape	No	No

[a] Failure of a stability augmentor.

ings with increasing turbulence was shown. Also, as turbulence increased, additional benefits from the improved yaw SAS are seen, because of the improved authority of the powered rudder over the present fleet system servo authority limits. At 10 fps rms lateral gust input, the increment in pilot rating is approximately two for EYD over MYD, one for improved yaw SAS over EYD, and an additional one half for the addition of a roll SAS to the improved yaw SAS.

Figure 3 shows rms sideslip angle data vs rms lateral gust input for the same conditions as Fig. 2. These data show the same trend that the pilot ratings would imply. The improved yaw SAS substantially reduces the sideslip response, especially at the higher turbulence intensity. These data show the results of rudder travel limitations which result in saturation of the EYD system. At 10 fps rms lateral gust input, the improved yaw SAS produces less than one half the rms sideslip angle response of the EYD. This result implies a significant reduction in airframe loads. Also, note that if the data lines were extended they would intersect the ordinate at a finite value. This finite value is indicative of the airplane output due to pilot control inputs required to accomplish the flight task.

Dutch Roll Criteria Study Results

Piloted simulation studies were conducted to assess the effects of Dutch roll frequency and damping on the airplane handling qualities. The studies utilized the analog program and the fixed-base point light simulation facility previously described.

The test plan was set up to determine the effects of Dutch roll frequency and damping on the B-52 handling qualities. The changes in Dutch roll frequency and damping were obtained by modifying the unaugmented airplane stability derivatives. The method utilized was as follows: 1) $C_{n\beta}$ was computed to achieve the desired Dutch roll frequency. 2) $C_{l\beta}$ was computed to maintain roll-sideslip ratio constant. 3) $C_{l_{\delta a}}$ was computed to maintain constant roll rate. 4) $C_{n_{\delta a}}$ was computed to maintain the proverse yaw characteristics constant. 5) C_{l_p} was computed to maintain a constant roll time constant. 6) C_{n_r} was computed to achieve the desired Dutch roll damping. 7) C_{l_r} was computed to maintain a constant spiral time constant.

The other parameters in the lateral-directional and longitudinal axes were held constant at values representative of the

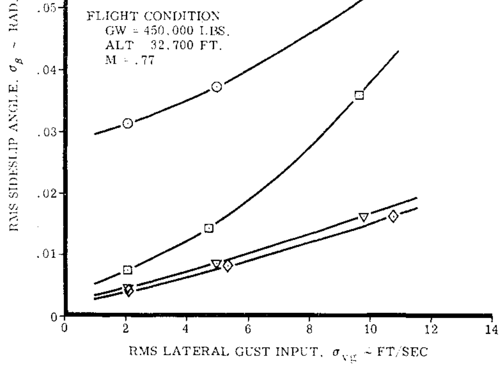

Fig. 3 Sideslip response with stability augmentation systems.

Dutch Roll Natural Frequency

$$\omega_d^2 \doteq \frac{\rho S U_o^2 b}{2 I_{zz}} C_{n_\beta}$$

Dutch Roll Roll/Sideslip Ratio

$$\left|\frac{\phi}{\beta}\right|_d \doteq - \frac{C_{\ell_\beta} I_{zz}}{C_{n_\beta} I_{xx} \sqrt{1 + \frac{1}{\omega_d^2 T_R^2}}}$$

Steady State Roll Rate

$$P_{ss} \doteq \frac{\rho S U_o b^2}{4 I_{xx}} C_{\ell_{\delta_a}} \delta_a T_R$$

Proverse Yaw Characteristics

$$\left(\frac{\omega_\phi}{\omega_d}\right)^2 \doteq 1 - \frac{C_{n_{\delta a}}}{C_{\ell_{\delta a}}} \frac{C_{\ell_\beta}}{C_{n_\beta}}$$

Roll Time Constant

$$\frac{1}{T_R} \doteq - \frac{\rho S U_o b^2}{4 I_{xx}} \left[C_{\ell_p} + \frac{C_{\ell_\beta}}{C_{n_\beta}} \left(\frac{2 I_{zz}}{m b^2} C_L - C_{n_p}\right)\right]$$

Dutch Roll Damping

$$2 \zeta_d \omega_d \doteq - \frac{\rho S U_o}{2 m} \left\{ C_{Y_\beta} + \frac{m b^2}{2 I_{zz}} C_{n_r} - \left(\frac{I_{zz}}{I_{xx}} \frac{C_{\ell_\beta}}{C_{n_\beta}}\right)\left(C_L - \frac{m b^2}{2 I_{zz}} C_{n_p}\right) \right\}$$

Spiral Time Constant

$$\frac{1}{T_s} \doteq - \frac{g}{U_o C_{\ell_p}} \left(\frac{C_{\ell_\beta}}{C_{n_\beta}} C_{n_r} - C_{\ell_r}\right)$$

Fig. 4 Approximate expressions, lateral—directional characteristics.

B-52H at high-altitude cruise. The pertinent short-period and phugoid characteristics are given in Table 2.

Table 2 gives the values of Dutch roll natural frequency and damping ratio selected for the simulation studies. Also shown on the table are the values of the proverse yaw characteristics (ω_ϕ/ω_d), the roll time constant (T_R), and the spiral time constant (T_S). The dynamic roll-sideslip ratio (ϕ/β), the steady-state roll rate (P_{ss}) and the time to achieve a bank angle of 30° were determined from analog computer time histories and were found to be approximately equal to 1.5, 16 deg/sec, and 3.1 sec, respectively. All pertinent lateral-directional parameters except the Dutch roll frequency and damping ratio were essentially constant for all of the conditions.

The equations shown in Fig. 4 are the approximate expressions that were used to determine the values of the aerodynamic derivatives that were utilized in the simulation. These equations gave a good approximation of the lateral-directional characteristics for the Dutch roll frequencies of one radian/second or greater. For the lowest frequency a "trial-and-error" procedure, utilizing root locus techniques, was used to determine the combination of derivatives that would give the desired results.

The combinations of Dutch roll frequency and damping were then tested in three levels of random turbulence. Three statistically independent sources of turbulence, all scaled to the same rms amplitude, were utilized to represent the vertical, the asymmetric vertical (roll), and the side gusts. Turbulence levels of 1, 5, and 10 fps were identified as low, medium, and high turbulence, respectively.

In an attempt to determine the significant characteristics in regard to Dutch roll, frequency, damping ratio, and time to damp were plotted vs pilot opinion. Typical data obtained, shown in Figs. 5 and 6, illustrate that at a given turbulence level, each subjects opinion depends primarily on the time to damp to half-amplitude ($\zeta\omega$). There is a significant difference between pilots in that for a given damping level and turbulence level there may be a difference of one rating point between the pilots. These phenomena are commonly referred to as "pilot set," e.g., each pilot tends to define his own optimum system and then all ratings are relative to this selected optimum. For each pilot, irrespective of turbulence level, there is no clearly definable effect of frequency when the time to damp to half-amplitude is constant. In some cases, it may appear that a low frequency is desirable, whereas in some other cases it appears that a high frequency would be more desirable; however, it is believed that these cases are due to data scatter.

It is also interesting to note that the turbulence has, as expected, an undesirable effect on pilot opinion. For these tests the pilot did not know what the turbulence intensity was and thus he had to grade the airplane on how precisely he could control it. The turbulence made the control problem more difficult and the pilot's opinions of what damping level was

Table 2 Simulated airplane characteristics[a]

Approximate		Exact						
ω_d	ζ_d	ω_d	ζ_d	ω_ϕ	ζ_ϕ	ω_ϕ/ω_d	T_R	T_S
	−0.1	2.02	−0.09	2.39	−0.06	1.18	1.33	101
	0.05	2.03	0.05	2.40	0.12	1.18	1.32	103
2.0	0.20	2.03	0.20	2.42	0.30	1.19	1.31	104
	0.35	2.03	0.35	2.43	0.46	1.20	1.31	105
	0.60	2.04	0.59	2.45	0.77	1.20	1.26	63
	−0.1	1.52	−0.09	1.79	−0.07	1.17	1.37	100
	0.05	1.51	0.06	1.80	0.14	1.18	1.35	103
1.5	0.20	1.54	0.20	1.82	0.31	1.18	1.34	104
	0.35	1.55	0.35	1.83	0.49	1.18	1.32	105
	0.60	1.48	0.54	1.85	0.78	1.25	1.29	104
	−0.1	1.04	−0.06	1.19	0.0	1.15	1.41	107
	0.05	1.05	0.08	1.21	0.18	1.16	1.40	108
1.0	0.20	1.06	0.22	1.22	0.35	1.16	1.40	109
	0.35	1.06	0.31	1.22	0.47	1.16	1.39	111
	0.60	1.06	0.58	1.25	0.81	1.18	1.36	112
	−0.1	0.60	−0.06	0.61	0.04	1.14	1.36	60
	0.05	0.57	0.09	0.70	0.24	1.23	1.37	120
0.5	0.20	0.59	0.27	0.72	0.50	1.21	1.40	118
	0.35	0.62	0.48	0.74	0.80	1.18	1.46	140
	0.60	0.67	0.67	0.76	1.09	1.13	1.35	160

[a] Short period: natural frequency = 1.13 rad/sec, damping ratio = 0.60. Phugoid: natural frequency = 0.051 rad/sec, damping ratio = 0.027.

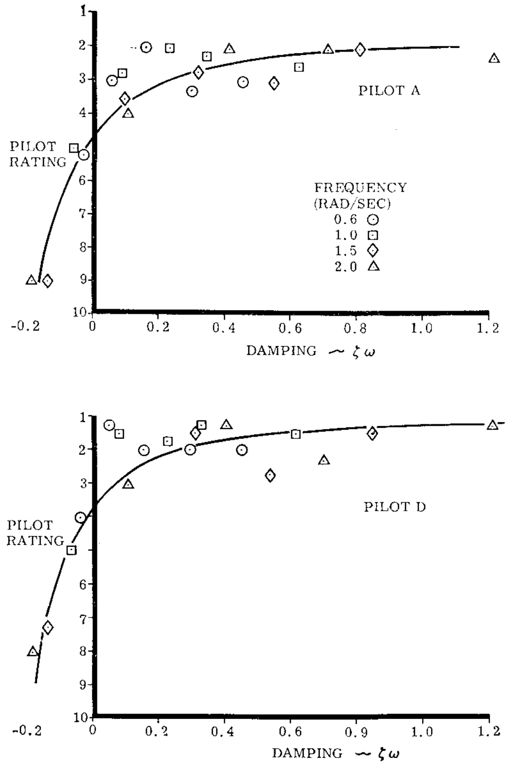

Fig. 5 Dutch roll evaluation—low turbulence.

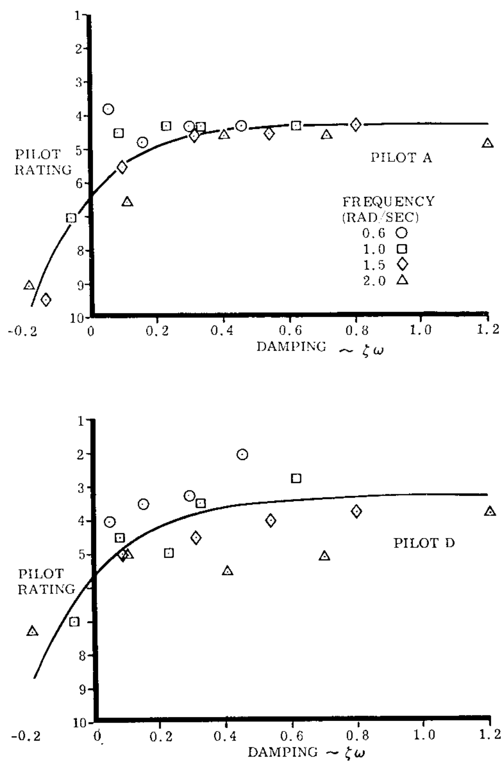

Fig. 6 Dutch roll evaluation—high turbulence.

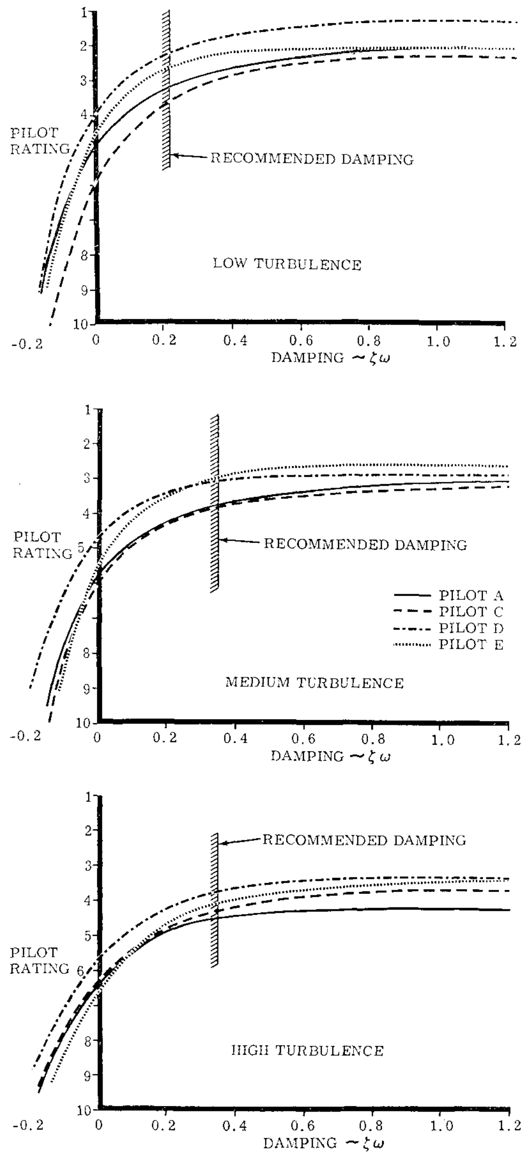

Fig. 7 Dutch roll evaluation—piloted simulation.

acceptable for emergency and nonemergency conditions changed. This trend points out the invalidity of using data obtained during calm or simulated calm conditions to establish emergency operation boundaries.

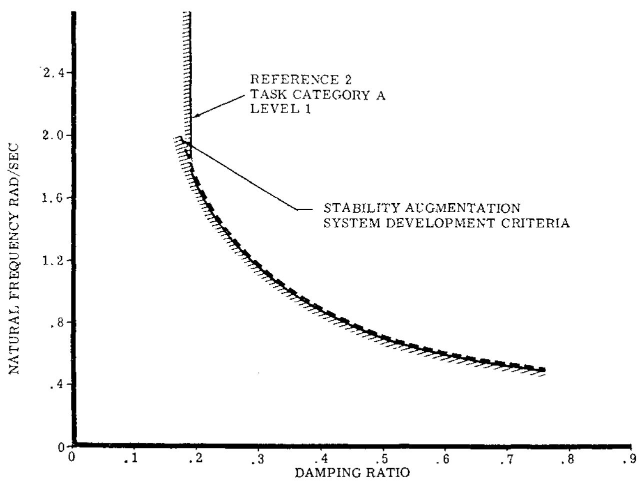

Fig. 8 Dutch roll criteria comparison.

Table 3 Comparison of prototype airplane with fleet B-52[a]

	Pilot rating			
	Degraded	No change	Improved	Much improved
Air refueling				
1) Yaw SAS on, pitch SAS on			3	1 2 4 5 6
2) Yaw SAS on, pitch SAS off		1	1 3 4 6	2 5
Dutch roll stability				
1) Low speed				1 2 3 4 5 6 7
2) Power approach (PA)			5	1 2 3 4 6 7
3) High-speed low level		5	2 4 6 7	1
Control power				
1) Elevator				
High speed			1 2 4 6	3 5
Low speed (PA)				1 2 3 4 5 6
2) Rudder				
High speed			2	1 3 4 5 6
Low speed (PA) (4 engines out)			5	1 2 3 4 6 7
Controllability in buffet				
1) High Mach				1 2 3 4 5 6
2) Initial stall	5		2	1 3 4 6

[a] Note: 1 through 7 refers to the seven pilots who performed this evaluation. The degraded rating under item 4 by pilot 5 refers to initial stall *warning* only. The airplane controllability in buffet was improved.

The summary curves in Fig. 7 show the individual pilot variation with respect to an average pilot rating. Also shown in Fig. 7 are the levels of damping that were set forth as desirable requirements. As can be seen, these levels of Dutch roll damping are sufficient to give satisfactory ratings for all conditions except the high-turbulence case, where two of the pilots did not feel that any level of damping was sufficient to provide satisfactory characteristics.

Even though the Dutch roll frequency does not affect the pilot's rating of the system provided the Dutch roll damping is constant, his verbal comments are dependent on the Dutch roll frequency. At Dutch roll frequencies of 1 rad/sec and above the pilot recognizes the Dutch roll oscillation and identifies it as such. At the lower frequency of 0.6 rad/sec, the pilot describes a heading problem. This is due to the fact that at the low frequencies the pilot can easily maintain a wings level attitude cancelling out the roll due to the Dutch roll. However, the pilot notes a heading change due to the Dutch roll oscillation. The snaking due to the Dutch roll oscillation is described as a heading problem.

A comparison of the criteria for medium and high turbulence utilized for the SAS development, with the proposed criteria of Ref. 2, is given in Fig. 8. Note that the criteria are in excellent agreement throughout the frequency range investigated. Both criteria specify a minimum damping value of $\zeta\omega = 0.35$, whereas Ref. 2 specifies minimum damping ratio and minimum frequency requirements in addition to the minimum damping requirement.

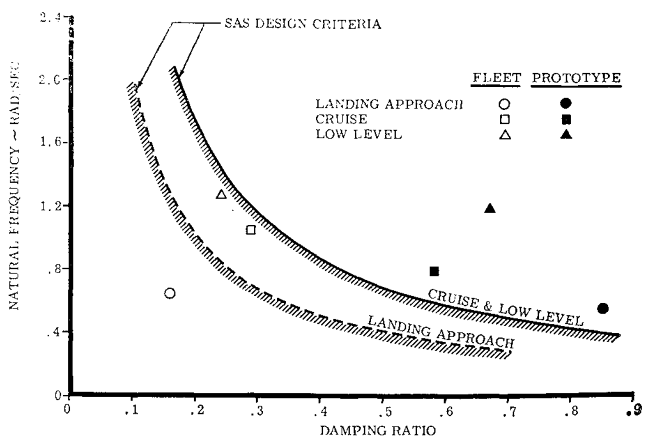

Fig. 9 Dutch roll damping flight test.

Flight Test Results

The improved stability augmentation system was flight tested in the January–August period of 1967 by both Boeing and Air Force test pilots. The summary curve, Fig. 9, shows the Dutch roll damping and frequency obtained with the prototype system as well as the Dutch roll criteria utilized for the SAS development. The pilots' comments during the test program were highly favorable in regard to the damping provided by the SAS in both the longitudinal and lateral-directional axis. The flying qualities of the airplane were also improved due to the addition of the hydraulically powered elevator and rudder, which improved the control authority as well as improving the preciseness of control input.

A summary checklist of the effect of the stability augmentation system on flying qualities is given in Table 3. This table depicts how each element of the new stability augmentation system affected the flying qualities.

The increase in rudder power was accompanied with a reduction in pedal force gradient which significantly improved the control harmony. The increase in elevator control power throughout the flight envelope also provided a significant improvement in the flying qualities. The improvement in control linearity also makes it possible to use more finesse in the landing pattern and at touchdown. Manual refueling performance is also improved because the pilot has a decreased tendency to overcontrol.

The improvement in Mach buffet characteristics can be attributed to two effects of the powered elevator; first, the effects of the tab feedback to the column has been eliminated which reduces the column shake due to Mach buffet; and second, the elevator deflection is maintained constant by the actuator eliminating an elevator condition associated with the present tab configuration which, in turn, causes a large g influence due to Mach buffet. The flight test results showed that the improved stability augmentation system would significantly improve the airplane structural characteristics (Ref. 1) and also enhance the airplane flying qualities.

References

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Simplified Design Equations for Buckling of Axially Compressed Sandwich Cylinders with Orthotropic Facings and Core

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In this paper, the linear theory for buckling of axially compressed orthotropic sandwich cylinders presented in Ref. 1 is investigated in terms of various material parameters utilizing a high-speed digital computer. Simplified design equations which approximate this theory are then presented. The accuracy of these simplified design equations is compared with that of the other available approximations using the analysis of Ref. 1 as a basis.

Nomenclature

a_i = coefficients defined in Eqs. (2)
c = core depth
D_x,D_y,D_{xy} = flexural rigidities of orthotropic sandwich shell
D_{qx},D_{qy} = transverse shear rigidities of orthotropic core material
E_x,E_y = Young's moduli of the facings in the x and y directions
G_{xy} = shear modulus of orthotropic facings
G_{xz},G_{yz} = shear moduli of the core in the xz and yz planes
h = $c + t$
K = dimensionless buckling parameter
L = axial length of cylinder
m = $m_1\pi/L$
m_1 = axial wave number
N_x = axial force resultant
N = $-N_x$
n = $n_1/2R$
n_1 = circumferential wave number
R = radius of middle surface of cylinder
S_x,S_y,S_{xy} = stretching rigidities of orthotropic sandwich shell
t = facing thickness
ν_{xy},ν_{yx} = facing Poisson's ratios
σ_{cr} = maximum fiber stress in facings at which buckling occurs in a cylinder subject to axial compression

Subscripts

cr = used to indicate critical buckling
rc = used to denote rigid core
x,y,z = coordinates on the cylinder middle surface in the axial, circumferential, and normal directions, respectively

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1. Introduction

REFERENCE 1 presented a linear general-instability analysis of an axially compressed sandwich cylindrical shell having both facings and core of arbitrary orthotropy.